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Frogs' Jumps: An Example of Using Computers as a Means of Empirical Validation

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Abstract

We present a sequence of didactical situations about ratio and proportion notions carried out with a fifth-grade group in an elementary school[16]*. The subject of the situations is frogs' jumps. For each frog there are three pieces of related data: the number of jumps done by the frog, the entire distance that it covers and the length of each jump. We included mostly tasks of ratio comparison (Which frog made the longest jump?) and tasks of missing value (Make the frogs' jumps the same length). To introduce the different situations and, above all, to give students a way to verify their predictions, an* ad hoc *program in Logo was designed. In this paper we describe the sequence of situations and the Logo program and analyse the role of computers in the didactical process.*

Keywords: Empirical validation in a didactical situation

In the learning process of constructivist approaches it is considered that subjects construct their knowledge through interaction with an environment that offers resistance, that implies some difficulties. In the case of mathematical learning, it is assumed that students construct their knowledge as tools that allow them to solve some problems, and only after a process of some length, involving situations without a context and generalisations, students acquire the knowledge as cultural, institutionalised knowledge (Douady, 1986).

This assumption about learning and mathematical knowledge has given rise to the study of a specific "environment", object of the student's interaction in order to propitiate the construction of specific knowledge as well (Brousseau, 1986). On the other hand, Douady (1986) notes the minimum number of conditions that a problem situation must satisfy to allow a process of autonomous search by the students. They are the following: 1) the student must clearly understand the goal that must be achieved and must be able to outline at least one procedure for a possible solution; 2) the student's knowledge must not be enough to solve the problem, at least to solve it in a systematic and optimal way; 3) the situation must give back information

16 Beside the authors of this paper, the following people participated observing and registering classes: Gabriela Gonzalez, Pilar Gonzalez, Marina Kriscautzky, Laura Resendiz. The Logo program was developed by Marina Kriscautzky and Patricia Martinez.

(feedback) to the student about his actions, it must let him know whether a specific solution is right or wrong; 4) the knowledge that we want students to get must be the optimal tool for solving problems at the level of the student's knowledge; 5) the problem needs to be formulated in different frames or registers (geometry, arithmetic, physics, ...).

We are interested in commenting on the third condition: the feedback that students receive from the situation, through empirical testing (Block, 1991; Brousseau, 1986). The possibility of testing by himself whether he attained the goal or not is a *sine qua non* condition for the student to work independently and to accept that a strategy he has chosen may be wrong or incomplete. So empirical testing is very important for the evolution of his strategies and of his previous knowledge. For a while this process may be viewed as a game, a situation that is being lived, by the student.

In the didactical experience we are presenting below, the computer program "The frogs' jumps" was the main form of empirical testing[17]. We will now briefly describe the didactical purpose of the situations and the characteristics of the program used, in order to analyse the role of the computer in the process.

1. The project: a didactical study of the notion of ratio

The sequence of situations is a stage of a research project about the acquisition of the notion of ratio in elementary school[18]. One of the hypotheses of this project states that explicit work with ratios expressed as pairs of whole numbers, prior to the expression of ratios as single numbers, not to mention fractions, would allow kids to better comprehend the relations that are involved in missing-value situations, and at the same time it could later become a foundation for constructing the complex notion of fraction as a linear application. In fact, results show that kids may know things about a relation such as "the frog covered a distance of 3 units in 5 jumps", without needing to know the result of the division 3 by 5, i.e., without quantifying the ratio involved in the relation by a single number. Children may know, for example, that those jumps are longer than those made by a frog that covered a distance of 6 units in 8 jumps, or that the jumps were as long as the ones made by a frog that covered a distance of 9 units in 15 jumps[19].

2. A sequence of situations: "the frogs' jumps"

This sequence involves six didactical situations with two types of task: comparison and missing value. For each kind of task we established several levels of difficulty, depending on the kind of relation between the numbers. We considered ratios[20] of numbers with common divisors (e.g., (2, 6)) and without (e.g., (3, 5)), but *in every case* the problem could be solved without using

17 Brousseau (1986) distinguishes three kind of testing: empirical (pragmatic), semantic and syntactic.

18 Block, D. *Estudio didáctico de la noción de razón en la escuela primaria* DIE-CINVESTAV IPN (PhD thesis, in progress).

19 There are several researches that are related to this subject and whose contributions have been important for this work. From different perspectives, we have considered for example Freudenthal (1983), Vergnaud (1988), Karplus (1983), Noelting (1980).

20 We use the following terms in the sense given by Freudenthal: an *internal ratio* is one that is established either between two amounts of jumps or between two amounts of units; an *external ratio* is one that is established between one amount of jumps and one of units.

fractions. We will now explain the two types of task, the level of difficulty and the procedures of solution they propitiated. At the same time we will explain the functioning of the program in comparison and missing-value situations.

3. Comparison situations

We explained to the children that two frogs had been competing in jumping and their task was to find out which frog had made the longest jump in each competition (the jumps' length was different each time). We told them that the winner was not the frog that covered a longer distance or made more jumps, but the one whose jumps were longer. We present the following activity as an example of the ones we carried out with the children.

First competition			
Green Frog		Purple Frog	
Whole distance	No. of jumps	Whole distance	No. of jumps
12 units	12 jumps	7 units	5 jumps

The frog that makes the longest jump is:________________________

Because:______________________________________

Children worked in pairs to answer which frog made the longest jumps and to argue why. Later, they went to the computer to verify the answer.

4. Levels of difficulty and expected procedures

- There is a common term. If both frogs cover the same number of units in the whole distance, for example *Green Frog* (5 units—5 jumps) and *Purple Frog* (5 u—3 j), students may consider only the number of jumps. The more jumps the frog makes, the shorter they will be. On the other hand, if the number of jumps is the same for both frogs, a longer distance implies a longer jump: GF (9 u—3 j) and PF (7 u—3 j).
- For one frog the distance covered and the number of jumps are the same, therefore, the length of the jump is a unit: GF (4 u—4 j) and PF (5 u—3 j). In this case, it is only necessary to check if the other frog's jump is longer or shorter than one unit.
- One frog's jump is shorter than one unit and the other's is longer than one unit: GF (2 u—3 j) and PF (10 u—6 j). In this case it is enough to compare in terms of the unit.
- The number of jumps or the distance covered is a multiple of its homologous: GF (4 u—2 j) and PF (6 u—4 j). If the *Green Frog* made another two jumps, that would be 4 jumps covering 8 units, so its jumps would be longer. If the ratio is three or greater, the problem is more difficult.
- The length of the jump made by each frog, or at least by one of them, is easy to calculate because the jump is a whole number of units or half a unit: GF (4 u—5 j) and PF (3 u—6 j). The jump made by the *Purple Frog* is half a unit. If the green frog had made jumps of the same length, the distance covered would be 2.5, but since the length is greater, the jumps are longer. This is another example: GF (6 u—2 j) and PF (15 u—3 j)
- The relation between the jumps or between the units is 3/2: GF (10 u—4 j) and PF (16 u—6 j). One can construct an "intermediate frog", *Frog* X (5 u—2 j), whose jumps are the same length than those of the GF. The *Purple Frog* makes three times more jumps than *Frog X*.

If it had covered three times more units, the distance would be 15 units, but since it covers one more unit, the jumps are longer.

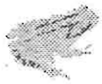
- None of the above relations are present: GF (5 u—4 j) and PF (4 u—3 j). In this case, one must obtain new pairs of data where both frogs make more jumps, for example GF (20 u—16 j) and PF (20 u—15 j). Here we find that the purple frog made one jump less than the green one with the same distance covered. Therefore, it makes longer jumps.

As we can see, the implied level of reasoning gets more complex each time. Work with both internal and external ratios is required. Although measures of fractions are involved, in no case is it necessary to use fractions (though some children will certainly try to start using them). Implicit fractions are handled through relations among whole numbers. Recall that the purpose is the development of the notion of ratio prior to its expression with fractions.

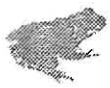
Modelling this situation in Logo. Two frogs of different colours (green and purple) and a start button are presented. Upon clicking on the start button, the screen clears and the frogs move to the left side.

start

The frogs must be pressed so that the distance of their jumps are graphed by the computer. Upon clicking the left button of the mouse on any of the frogs, the following questions appear:



What is the distance the frog will cover?
How many jumps does it need?



As soon as the last question is answered, the frog covers the indicated distance in the indicated number of jumps without giving the jumps length. For example, if it is stated that the green frog covers 50 units and makes 5 jumps and the purple frog covers 40 units and makes 2 jumps, the computer shows the following:

---------|----------|----------|----------|----------|

--------------------|--------------------|

When both distances covered appear, the student can check if his answer was correct. The Logo procedures for this activity are in *appendix 1*.

5. Missing-value situations

There are four frogs of different colours. The data given for one of the frogs includes the distance covered and the number of jumps. For the other frogs only one piece of data is given, or none at all. The task now is to look for the missing numbers in order to get every frog to make jumps of the same length as the model.

Activity 2

The jumps of the frogs must be of the same length. Find the missing values.

Frog	Whole distance	No. of Jumps
Green frog	12 units	3 jumps
Purple frog	___ units	9 jumps
Blue Frog	24 units	___ jumps
Red frog	___ units	5 jumps

Children can verify the results on the computer. Because of lack of space we have not described the difficulty levels but they may be inferred from those given in the comparison situations.

Modelling this situation in Logo. In this situation the program involves four frogs of different colours and a start button. Upon pressing the start button, the screen clears and the green frog covers a certain distance in a determined number of jumps.

Upon clicking on each frog the computer asks the following questions:

What is the distance the frog will cover?
How many jumps does it need?

Once these have been answered, the selected frog jumps the distance corresponding to the data entered. For example, if the green frog covers 12 units in 3 jumps and for the purple frog we answer that it would cover 27 units in 9 jumps, the frogs will do the following:

----I----I----I

---I---I---I---I---I---I---I---I---I

When the children look at what the frogs have done on the screen, they can see that the jumps are not the same length, but the program does not give an answer, so they have a chance to change the missing value and try to find a new number. If the next time they answer that the purple frog covers 36 units in 9 jumps, they will see that the jumps are the same length.

----I----I----I

----I----I----I----I----I----I----I----I----I

Although it is evident on the screen whether the jumps are the same length or not, we included another stimulus: a smiling frog if the jumps have the same length or a sticking out frog if they are not equal. The Logo procedures for this activity are in *appendix 2*.

6. Considerations about using computers in the classroom

The purpose of this program was to have a tool, first for introducing the situations, and second for verifying the results obtained by the children. Regarding the former, by using the program and being able to visualise the effects of the variables involved (e.g., by seeing how upon increasing the number of jumps the lengths get shorter and upon increasing the whole distance they get longer) the children could better understand the activity instruction and establish basic relations among data.

Regarding the latter, the program has the characteristic of giving visual feedback in relation with the proposed data without suggesting any procedure to get the result.

In the case of the comparison activities, when observing which of the two frogs made the longest jump, the children can immediately find out if they are right or wrong. They do not receive any other information to tell them what they have to take into account to be successful the next time. By analysing the relations within the data, the children will anticipate the frog's behaviour on the computer. This process takes place when performing other similar activities that involve testing hypotheses and sharing findings.

In the missing-value activities, the first thing that children learn from checking an erroneous answer, for example one where the jumps are longer than the model frog's, is that to increase the length of the jump they have to increase the whole distance. Later, they also find that they can get the same effect by reducing the number of jumps.

In such situations, the program gives visual feedback about the result (the jumps are or are not the same length); besides, it allows the children to consider how many units or jumps they have to add or to subtract in order to get the same length for the frogs' jumps.

However, there is a caveat: if we let children stay by the computer or retry their results indefinitely, they may attain the goal little by little, but without getting to establish the necessary numerical relations. This can be useful the first time, but we have to avoid it on further occasions.

On the other hand, if in any case the children move on to empirical testing before due time, it might have a negative effect: avoiding to test results without arguing them (semantic validation). These considerations show the necessity of distinguishing different moments in the didactical process which allow different ways of using computers. Let us explain.

7. Alternative ways of using the computer to test results

The school where we did our field work didn't have computers available, so we did the work with two computers that we took to the site. We were interested in taking advantage of this tool in these conditions, because they are common in most of the schools in our country. Nevertheless, the scarcity of computers prevented students from using computers to obtain results without having thought about the problem which was presented to them.

It was necessary then to alternate two ways of testing: in some cases it was collectively (more often in the first three sessions), in other cases two students worked together to test their results. Below we present the ways in which we worked, the possibilities and difficulties in each case, and some examples.

7.1 Collective testing

At the end of the each activity we organised a collective discussion of results, several children gave and defended their results, often explaining the procedure that they had used to obtain them. After listening to the explanations some children decided to change their results. Others were not convinced by what they had heard. They asked to run the program and see what the frogs actually did. Only when the discussion was over did the teacher run the program to verify results proposals. Let us see an example with the following data.

Green Frog		Purple Frog	
Whole distance	**No. of jumps**	**Whole distance**	**No. of jumps**
28 units	5 jumps	36 units	5 jumps

The teacher asked the students who among them thought that the green frog would win the competition (four children raised their hand, the rest of the group bet on the purple frog). The teacher asked the first group of children to argue for their answer. It was difficult for the children to explain why they thought a frog made a longer jump. After having done several activities to the frog competition program and after having listened to different explanations, the students began to give better arguments.

In this example two children explained why they thought the green frog had won the competition.

Francisco said *"because there are 28 jumps, the whole distance is 28 and it makes 5 jumps. The whole distance is divided by five and then the jumps are bigger"*. Rachid read from his notebook: *"The green frog makes somewhat longer jumps than the purple one"*. Trying to explain his argument he realised that he was wrong and said about the purple frog *"Ah, the whole distance is 36 units, the distance covered is longer than for the green one, the jumps must be longer"*.

Several children volunteered to argue why the purple frog would have won the competition. Gabriel said: *"there are 36 units. Five divides 36 units and that is going to make the jumps of the frog longer. They are going to be longer"*. Beatriz said: *"The number of jumps is the same for both frogs but the whole distance is bigger* (pointing to the 36 units). *I mean that the jumps must be longer"*.

The teacher decided to run the program so the children could watch the frogs' actions. The children who had supported the purple frog cheered, the others seemed disappointed, but as they watched the developments, they seemed to better understand the activity.

In the case of the missing-value activities, the collective discussion was about the proposed numbers for the missing values. The teacher wrote them on the blackboard and asked children to argue which was right or wrong. After these arguments the program was run so that the children might watch the actions of the frogs. Let's see an example.

Several children proposed the data for the following frogs so that they could make jumps of the same length as the green one.

Frog	**Whole distance**	**No. of Jumps**
Green frog	12 units	3 jumps
Frog 1	14 units	4 jumps
Frog 2	48 units	12 jumps
Frog 3	30 units	6 jumps

Before running the program, the teacher asked the children to comment if at first sight they could find something wrong considering that all the frogs' jumps must have been the same length.

One child said that the whole distance of the *Frog 1* was wrong and proposed to change it to 16 because 4x4=16. A girl proposed to change the whole distance of the *Frog 3* to 24 because 6x4=24. The teacher wrote these answers next to the previous ones and then ran the program in the order in which the proposals were given. When he tested the *Frog 1* data with 14 units in whole distance, at the frog's first jump the children said that it was not the same length as the green frog's jump. They proposed to test again with 16 units in the whole distance so that they could see at that moment that the jumps were the same length. They did the same process for *Frog 3*.

7.2 Couple testing

The time invested into solving a problem is different for each child. It mainly depends of the procedure used to find the solution. We decided to put two computers in two different places in the classroom. When the children were one with the activities, they had the opportunity to test their results on the computer. If they saw that their answers were wrong, they had the chance to return to their seats, analyse the problem once again, propose another result and retest. If they arrived at the correct answer, the children were given new activities while the others finished the previous assignment.

We had a difficulty in this modality. Every child wanted to test their results in the computer even when they were sure the answer was right. This situation became a problem of long queues in front of the computers and resulted in the loss of a great deal of time and the anticipated diffusion of right results.

We had to generate strategies for limiting the use of the computers and making it more efficient. We asked one person in the research team to speed up the testing process (kids dictated data but the adult typed them in to the computer) and the students to only test one of their results, the one they were not sure about. Considering that there were kids who solved the problems quickly, we suggested that if they were sure of their answers, they need not test them but could proceed to working on the more difficult problems that we had prepared. This way those students who had more difficulty solving problems also had more opportunities to test their results on the computer. Let's see what Miguel and Francisco did in one missing-value activity.

	Whole distance	**Jumps number**
Green frog	12 units	3 jumps
Purple frog	36 units	9 jumps
Blue frog	24 units	6 jumps
Red frog	23 units	5 jumps

The students were able to calculate the distance that the purple frog covered by observing that it jumped three times more than the green frog. They multiplied 3×3=9 and 12×3=36. In the case of the blue frog, students observed that 24 was two times 12. Therefore they doubled the jumps made by the green frogs: 3+3=6.

By contrast, for the red frog it is not possible to use this kind of relation (internal ratio conservation) as in the learning process of the ratio concept. The children saw that the red frog made **one jump less** than the blue frog. Therefore they thought that its whole distance had to be 23 units, one unit less than the blue frog (additive strategy). They decided to test this last result on the computer because they were not sure that it was right. They were amazed to see that the jumps were not the same length. They were a little longer. They said that the whole distance data should be smaller. Taking advantage of the teacher's distraction, they stayed at the computer testing their findings. They tested a distance of 22, then 21 and finally 20.

8. Final remarks

We pointed to the need of analysing the specific goals of different moments in a didactical process and the search for better ways of using the computer in each moment. We saw the role and importance of giving students a clear introduction, so that they may better understand what is expected of them. The use of the computer when a first hypothesis is made. and the importance of delaying its use until students have had the opportunity to defend and deliberate their results, were also stressed as important steps in the whole didactical process. These steps integrated the use of the computer in the learning process and enriched it. The computer did not control the learning process, nor limit the involvement of students in the process of personal research of solution strategies. They prepared arguments to defend their results and debated the findings of their fellow students.

On the other hand, this experience helped us see that we need to find better ways of organisation so as to avoid an out-of-control situation due to the scarcity of computers in the classroom. It is common for computers to be in short supply in Mexican public schools.

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Appendix 1. Comparison situations

(The procedure **start** clears the screen and the frogs move to the left side)

```
To start
cls
t1, pu setpos
[-300 0]
t2, pu setpos
[-300 -30]
end
```

(The procedure **green2** asks about the whole distance and the number of jumps of the green frog. Then with the entered data the jumps are graphed on the screen. There is another procedure with the same instructions for the purple frog)

```
To green2
question [What
is the distance
the frog will
cover?]
asign [dist
answer]
question [¿how
many jumps does
it need?]
asign [jump
answer]
green
end
```

```
To green
t1, pu setpos
[-300 0]
pd fclolor 15
repeat :jump
[fd :dist *10
/:jump lt 90 fd
10 bk 10 rt 90
wait 10] pu fd
30
end
```

Note: We used microworld Logo, Computer systems Inc. 1996, version 1.5

Appendix 2. Missing-value situations

(This program involves four frogs of different colours. The procedure **start** clears the screen and the frogs move to the left side

```
To start
cls t1, pu setpos [-300 0] t2, pu
setpos [-300 -40]
t3, pu setpos [-300 -80] t2, pu
setpos [-300 -120]
green
end
```

(The green frog makes three jumps of three units each one)

```
To green
t1, pu setpos [-300 0] frumbo 90
pd setcolor 15
repeat 3 [fd 4 * 10 lt 90 fd 10 bk
10 rt 90 wait 10] pu fd 30
end
```

(We present the three procedures for the purple frog. They are the same for the two other frogs. **purpleask** asks the whole distance and the number of jumps for the purple frog. **purpleok** graphs the jumps if they are the same length as the green frog's and then we can see a smiling frog. **purplewrong** graphs the jumps if they are not the same length as the green frog's and then we can see a frog sticking out)

```
To purpleask
question [What is the distance the frog
will cover?]
asign [dist answer]
question [¿how many jumps does it need?]
asign [jump answer]
if :dist / :jump = 4 [purpleok]
purplewrong
end
```

```
To purpleok
t2, pu setpos [-300 -40] seth 90 pd
setcolor 95
repeat :jump [fd :dist * 10 / :jump lt
90 fd 10 bk 10 rt 90 wait 10] pu fd 30
t1, repeat 10 [setfig 15 wait 1 setfig
28, wait 1]
stop
end
```

```
To purplewrong
t2, pu setpos [-300 -40] seth 90 pd
setcolor 95
repeat :jump [fd :dist * 10 / :jump lt
90 Fd 10 bk 10 rt 90 wait 10] pu fd 30
t1, repeat 10 [setfig 40 wait 1 setfig
41, wait 1 setfig 42, wait 1]
stop
end
```